

Asteroid Redirect Mission

Background

The current probability of asteroid 99942 Apophis (2004 MN4) NEO (Near Earth Object) hitting Earth in April 2029 is near zero. (SentryWeb1). Following its discovery in 2004, the initial risk assessment by NASA-JPL's impact monitoring service, Sentry, was 1/300 (Delbo 2007). This was the highest impact risk ever reported since impact risk monitoring began. Since then with additional observations and the use of pre-discovery images the impact risk has been reduced so that the Sentry Risk Table does not list Apophis as a collision risk on 13 April 2029. The earliest risk now appears in 2060 with a probability of 5.7×10^{-6} or 1 in 175,000.

We do however know that large asteroids have struck Earth in the past and triggered major species extinction events. The most well known is the *KT event* 65 or so million years ago when an object estimated to have been 10Km in size hit the Yucatan peninsula on the gulf of Mexico and precipitated the demise of the dinosaurs and many other species.

At the other end of the scale small asteroids and large meteoroids have struck Earth in recent history. In 1908 a small comet or asteroid estimated to be between 30 and 50m in diameter exploded above a remote Siberian river Tunguska in what is called the Tunguska Event. The airburst flattened about 2500 km² of Siberian forest and is estimated to have released the energy of about 20 megatons of TNT. As recently as 2013 a superbolide (another name for a very large meteoroid) exploded 30km above the Russian city of Chelyabinsk while entering the atmosphere at a shallow angle. The airburst is estimated to have been caused by a body of about 17 to 20 m in diameter of mass 10,000 tons releasing the energy of 0.5 MT of TNT (or 2×10^{15} J) (Brown 2013). The explosion shattered a large proportion of all windows in the city injuring about 1500 people mainly by flying glass. One study (Lobanovsky 2014) put the size of this object at 180m and its mass at 1.8 megatons.

The frequency of asteroid/comet impacts on Earth have been estimated of Tunguska/Chelyabinsk size (30-50m) as one in 150-400 years, 100m size as one in 10,000 years, 1km size as one in several hundred thousand years (NEOsearch), 10km size one in several 10s of millions of years. Most of these estimates range over an order of magnitude depending on which authority or method was used to estimate it. Apophis size asteroids are estimated to strike Earth at the rate of 1 in 63,000years (Asher 2005).

While impacts of large asteroids that have global effects are extremely rare, the scale of the resulting destruction and threat to life on Earth and civilization is so great that public concern in the late 1990s led to the establishment of several programs by NASA, the IAU and internationally to find and monitor dangerous objects in the solar system. (NEOsearch, NEOdys, SpaceGuardWeb).

Systematic NEO searches began in 1998 when the US congress gave NASA the congressional directive of locating to 90% confidence level all PHAs (Potentially Hazardous Asteroids) which were judged to be asteroids with diameters > 1km. NASA then commissioned Catalina Sky Survey with three observatories participating (Catalina, Mt Lemmon and Siding Springs). (CSSweb). That goal has now been achieved and NASAs mandate has been extended in 2005 to find all PHAs > 140m (NASAweb). Current surveys in this program include Pan-STARRS (PAN-STARRSweb) and NEOWISE. Sadly SSS (Siding Springs Survey with Robert McNaught) has dropped out due to withdrawal of funding last July. (private communications).

The Sentry Risk Table is part of NASA's Near Earth Program site (SentryWeb1) which lists all current Potentially Hazardous Asteroids (PHAs) including their ranking on the Torino Scale. The Torino Scale ranks PHAs using a number between 0 and 10 with 0 denoting no hazard and 10 denoting certain collision with globally catastrophic destruction. The Torino hazard number takes into account the PHAs probability of collision in the next 100 years and the energy released on impact. (Table 1.) (SentryWeb2)

THE TORINO SCALE

Assessing Asteroid/Comet Impact Predictions

No Hazard	0	The likelihood of collision is zero, or is so low as to be effectively zero. Also applies to small objects such as meteors and bolides that burn up in the atmosphere as well as infrequent meteorite falls that rarely cause damage.
Normal	1	A routine discovery in which a pass near the Earth is predicted that poses no unusual level of danger. Current calculations show the chance of collision is extremely unlikely with no cause for public attention or public concern. New telescopic observations very likely will lead to re-assignment to Level 0.
Meriting Attention by Astronomers	2	A discovery, which may become routine with expanded searches, of an object making a somewhat close but not highly unusual pass near the Earth. While meriting attention by astronomers, there is no cause for public attention or public concern as an actual collision is very unlikely. New telescopic observations very likely will lead to re-assignment to Level 0.
	3	A close encounter, meriting attention by astronomers. Current calculations give a 1% or greater chance of collision capable of localized destruction. Most likely, new telescopic observations will lead to re-assignment to Level 0. Attention by the public and by public officials is merited if the encounter is less than a decade away.
	4	A close encounter, meriting attention by astronomers. Current calculations give a 1% or greater chance of collision capable of regional devastation. Most likely, new telescopic observations will lead to re-assignment to Level 0. Attention by the public and by public officials is merited if the encounter is less than a decade away.
Threatening	5	A close encounter posing a serious, but still uncertain threat of regional devastation. Critical attention by astronomers is needed to determine conclusively whether or not a collision will occur. If the encounter is less than a decade away, governmental contingency planning may be warranted.
	6	A close encounter by a large object posing a serious, but still uncertain threat of a global catastrophe. Critical attention by astronomers is needed to determine conclusively whether or not a collision will occur. If the encounter is less than three decades away, governmental contingency planning may be warranted.
	7	A very close encounter by a large object, which if occurring this century, poses an unprecedented but still uncertain threat of a global catastrophe. For such a threat in this century, international contingency planning is warranted, especially to determine urgently and conclusively whether or not a collision will occur.
Certain Collisions	8	A collision is certain, capable of causing localized destruction for an impact over land or possibly a tsunami if close offshore. Such events occur on average between once per 50 years and once per several 1000 years.
	9	A collision is certain, capable of causing unprecedented regional devastation for a land impact or the threat of a major tsunami for an ocean impact. Such events occur on average between once per 10,000 years and once per 100,000 years.
	10	A collision is certain, capable of causing a global climatic catastrophe that may threaten the future of civilization as we know it, whether impacting land or ocean. Such events occur on average once per 100,000 years, or less often.

Table 1. (Credit NASA Near Earth Object program site)

The US Congress's NASA Authorization Act of 2005 that extended the NEO search program to asteroids of 140m diameter also directed NASA to analyse possible alternatives for the diversion of objects on likely collision course with Earth. This has kicked off a plethora of scientific and not so scientific studies of strategies for asteroid hazard mitigation. (NSSweb).

Supposing asteroid 99942 Apophis is scheduled to hit Earth with certainty the Torino Hazard Scale is listing it at level 9. This project outlines a strategy to save Earth from the resulting catastrophe.

Asteroid (99942) Apophis

Apophis orbit diagram

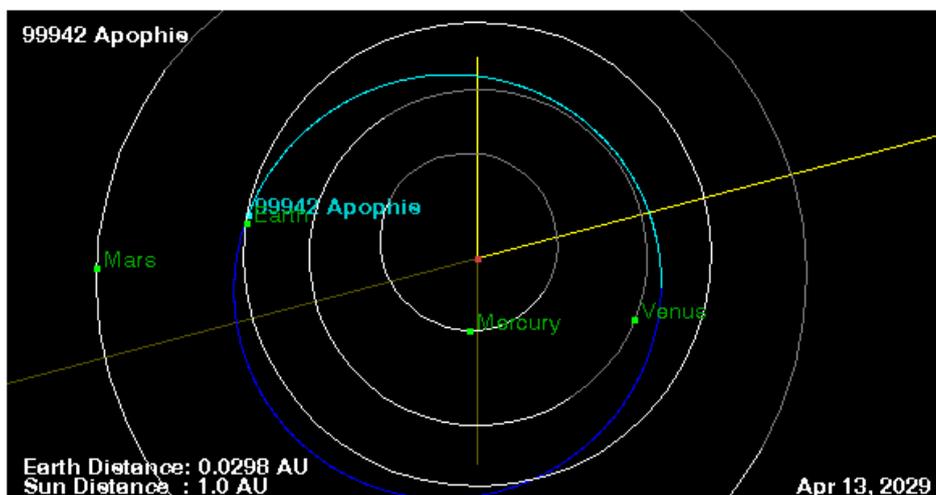


Fig 1. Orbit diagram showing Earth and Apophis on Apr 13, 2029 (Credit JPL)

Orbital Elements

Element	Value	Uncertainty	Units
Eccentricity (e)	0.1910795219761208	5.1343e-09	
Semi-major axis (a)	0.9222788903126814	5.3344e-10	AU
Inclination (i)	3.331290217797427	6.5897e-07	deg
Long. Asc. Node (Ω)	204.4571896287176	3.3046e-05	deg
Arg. Of perihelion (ω)	126.3936405969297	3.2426e-05	deg
Mean anomaly (M)	215.5399783483177	3.9882e-06	deg
Time of perihelion(t_p)	2008-Apr-14.81862480 2454571.31	3.6113e-06	Date JED
Perihelion (q)	0.7460502808230671	4.7924e-09	AU
Aphelion(Q)	1.098507499802296	6.3536e-10	AU
Mean motion(n)	1.112783484416944	9.6543e-10	deg/d
Period (T)	323.513068841623	2.8067e-07	d

Table 2. (credit JPL Small Body Browser, retrieved 24 Oct 2014)

Physical Parameters

Parameter	Value	Units	Reference
Absolute magnitude	19.7	mag	Delbo, Cellino and Tedesco (Icarus 2007)
Magnitude slope	0.25		Delbo, Cellino and Tedesco (Icarus 2007)
Diameter	0.325	Km	T. Muller (2013)
Rotational period	30.4	Hours	Behrend (2005)
Geometric albedo	0.23		T. Muller (2013)
Mass	4×10^{10}	kg	JPL

Table 3. (credit JPL Small Body Browser retrieved 24 Oct 2014)

99942 Apophis (2004 MN4) Earth Impact Risk Summary			
Torino Scale (maximum)	0	V_{impact}	12.62 km/s
Palermo Scale (maximum)	-3.32	V_{infinity}	5.85 km/s
Palermo Scale (cumulative)	-3.19	H	19.7
Impact Probability (cumulative)	5.7e-06	Diameter	0.330 km
Number of Potential Impacts	12	Mass	4.0e+10 kg
		Energy	7.5e+02 MT
Analysis based on 13 radar delay, 7 Doppler, and 3987 optical observations spanning 3318.0 days (2004-Mar-15.126289 to 2013-Apr-15.15729)		all above are mean values weighted by impact probability	

Table 4. Summary of impact risks (credit NASA NEO Program, Sentry Risk Table)

If Apophis collides with Earth the energy released will be in the order of 750 megatons of TNT. (SentryWeb2) In other words it would be equivalent to about 50,000 Hiroshima size nuclear bombs.

The Earth Impact Effects calculator (ImpactEffectsProgram) lists the following effects of impact by an Apophis size object at 10km from ground zero:

Energy before impact:	1250 Megatons TNT
Impact frequency	One impact per 9.1×10^4 years
No Global Effects	
Starts to break up in upper atmosphere at	44.7km
Fragments hit ground over	488m ellipse
Initial crater	4.65km diameter
Initial depth	1.64km
Final crater	5.71 km
Final Depth	500m
Melted or vaporized target volume	0.0479 km ³
Seismic shaking Richter Scale magnitude	6.7
Ejecta arrives 45.2 seconds after impact (fine dust, some large fragments)	
Average Ejecta thickness	4.16 meters
Mean Fragment diameter	15.8 meters
Air blast arrives	30 sec after impact
Peak overpressure	8.49 bars
Max wind velocity	696m/s
Damage description (
Multistory wall-bearing buildings will collapse. Wood frame buildings will collapse completely. Highway truss bridges will collapse. Highway girder bridges will collapse. Glass windows will shatter. Up to 90% of trees blown down, rest stripped of leaves. (ImpactEffectsProgram)	

Deflection Strategies

Most strategies for deflecting asteroids rely on having sufficient lead time so that a small amount of energy is required to modify the target asteroids orbit enough to miss the Earth on the day predicted for the collision and possibly for all time. The strategies fall into two broad categories: kinetic deflection and low thrust deflection. Kinetic deflection methods impart sufficient impulse and ΔV to instantly modify the asteroids orbit to miss Earth. Low thrust methods apply a very small continuous force over a long time to accumulate the required change in the orbit. The required change in the orbit in all cases must be at least one Earth radius at the point of intersection of the asteroids and the Earth orbit. Here are some examples of the proposed strategies.

- ***Nuclear explosion*** . A nuclear bomb of sufficient energy is exploded on or near the asteroid at the correct point in its orbit to minimize the energy required. The explosion blows off a large amount of material from the asteroids surface which imparts a reactionary impulse in the opposite direction as per Newton's 3rd law. The very real problem with this method is that the asteroid will shatter and parts of it will fly off in unpredictable directions and may cause multiple impacts on Earth instead of none. (Ahrens & Harris 1992). Despite its dangers this method seems to be still the most reliable and practical of all.
- ***High velocity kinetic impactor***. A large mass of sufficient velocity and mass is driven into the asteroid to alter its orbit to miss Earth. Unfortunately the relative velocities and masses required that will do the job on even a small asteroid is not practical with current technology (10 tons, 10km/s). McInnes (2004) proposed a design where projectile of modest mass was placed into a retrograde solar orbit using a low thrust solar sail driven spacecraft that after sufficient time to achieve the desired orbit collides with the asteroid with a velocity of 60km/s. There are major uncertainties in this scheme.
- ***Using a smaller asteroid as projectile*** . This method relies on a two stage kinetic impact method. A small asteroid is found that can be diverted by existing impactor technology then by gravity assist flyby to hit a small body on a similar orbit to the target, which then impacts on the offending asteroid to make it harmless. This method appears to be very problematic as it requires too many issues to be solved before it can be considered to be practical. (Ledkov 2014)
- ***Gravity Tractor*** . A (heavy) spacecraft is used to hover very close to the asteroid using the gravitational attraction between the asteroid and the spacecraft for a very small change (a few hundred meters relative to the predicted path) in trajectory. This method is proposed as part of the AEMP (Aposhis Exploration and Mitigation Platform) to shift Apophis out of a small keyhole as it passes Earth in 2029 and may impact Earth in 2036. AEMP is a collaboration by a number of groups including the TAMU Apophis Study group At the Texas A & M University, with the support of NASA Ames Research Center. (AEMPweb, Hyland 2009)

- **Solar Sail** This method would use a sufficiently large solar sail tethered to the asteroid to provide enough thrust to modify the orbit after years of continuous application. The main problem with this method is the difficulty of maintaining the sail in the correct attitude for the thrust vector while tethered to a rotating asteroid.
- **Surface sublimation methods** . First proposed by Melosh (1993), this method involves an array of carefully aligned parabolic mirrors of sufficient size to continuously vaporize a spot on the asteroid surface producing a jet of vaporized material to provide thrust. The obvious problem would be how to construct a 0.5km dish and maintain it accurately pointed on a spot on the surface for years.
- **Albedo modification** . This method proposes to paint the asteroid to change its albedo. Changing the albedo of a rotating body such as a small asteroid also changes the strength of the Yarkovsky Effect. The Yarkovsky Effect is a non-gravitational orbital perturbation caused by non-symmetrical re-radiation of solar heating causing a net thrust either in the prograde or retrograde direction. The effect is very small and changes to the albedo cause an even smaller change to non-gravitational perturbation. It could be enough, however, to push Apophis out of a deadly trajectory keyhole that may result in impact with Earth in 2036. (Margulieux 2010)

Mission Design

I have decided to design a deflection mission around a solar sail. This method appears to be the most cost effective and possibly quite doable. It is assumed that we have 15 years to plan, prepare and execute the mission resulting in changing Apophis's orbit enough to miss the Earth on 13th of April 2029. This requires shifting the path by 2 Earth radii from the impact point.

In short, the mission involves sending a robotic spacecraft carrying all the necessary materials and equipment to rendezvous with the asteroid, ~~land on~~ Apophis, deploy the pre-constructed and packaged, adjustable frame solar sail, dock with the deployed sail and adjust and maintain the sail assembly in correct alignment with the sun for about 10 years.

The elements of the design are: Solar sail and tethering system, trajectory to Apophis and launch vehicle selection.

- **Solar Sail Design**

Assuming we have 12 years of thrust from the sail, the continuous acceleration required to deflect Apophis by 2 Earth radii is given by:

$$a = 2R_e/3t^2 \text{ (Melosh 1993)}$$

where R_e is Earth's radius(m) = 6.378×10^6 m,

$$t = 12 \times 365.25 \times 24 \times 3600 = 3.787 \times 10^8 \text{ s.}$$

$$a = 2.965 \times 10^{-11} \text{ ms}^{-2}$$

Solar radiation falling normal to a perfectly reflecting surface is given by:

$$P_{\text{reflect}} = 2 F_{\text{solar}}/c$$

Where P is pressure Nm^{-2} , F_{solar} is solar flux at 1 AU = 1366 Jm^{-2} , c is speed of light $3.0 \times 10^8 \text{ ms}^{-1}$

$$= 9.1 \times 10^{-6} \text{ Nm}^{-2} \text{ (9.1 } \mu\text{Pa)}$$

Force required to accelerate Apophis by $2.965 \times 10^{-11} \text{ ms}^{-2}$

$$F = ma \quad m \text{ is mass of Apophis} = 4.0 \times 10^{10} \times 2.965 \times 10^{-11} \sim \mathbf{1.2 \text{ N}}$$

$P = \text{Force/Area}$ then $\text{Area} = \text{Force/Pressure}$

Area of sail needed to accomplish the mission in 12 years therefore = $1.2/9.1 \times 10^{-6} = 1.3 \times 10^5 \text{ m}^2$

The sail must be about $360 \times 360 \text{ m}$.

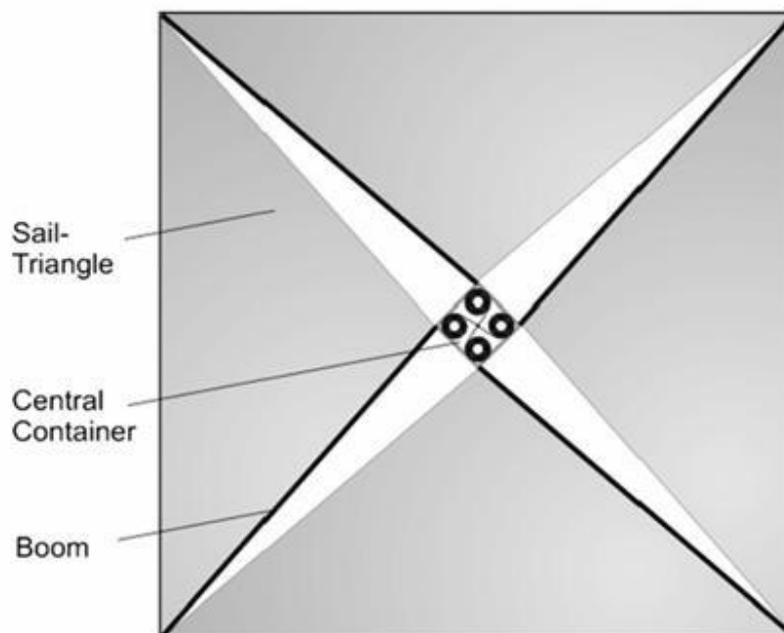


Fig. Diagram of sail showing major components (credit Block, J. German Aerospace Center)

We will use the design by the German Aerospace Center as published by Block, J. (2010) for a project supported by ESA (McInnes 2001). Using materials suggested by McInnes (2001) the weight budget for the sail and frame:

Sail film $3\mu\text{m}$ $10\text{g}/\text{ms}^{-2}$, Deployable CFRP (Carbon Fiber Reinforced Polymer) booms $150\text{g}/\text{m}$

Booms (1020m)	153kg
Sail material ($130,000\text{m}^2$)	1300kg
Mechanisms	100kg
Sail total	1553kg
Tethering system (est.)	1000kg
Packaging	500kg
Spacecraft systems	100kg
Propulsion + propellant	500kg
Total spacecraft mass	3653kg

This is the payload in Earth orbit that will rendezvous with Apophis. We need now to work out the capacity of the propulsion system to take this payload to Apophis.

- **Trajectory Design.**

Apophis is currently on an Earth crossing orbit with semi-major axis about 0.92AU , slightly less than Earth (1AU) therefore it should not require a lot of energy to transfer a spacecraft from LEO to Apophis's orbit.

(Note: the following calculation assume that Earth and Apophis have circular orbits and 0 inclinations to the ecliptic. In reality both orbits are elliptical and the plane of Apophis's orbit is inclined more than 3 degrees to the ecliptic. Accurate calculations using the elliptical orbital elements are outside the scope of this course)

To calculate the conventional propulsion system and propellant required to take the solar sail package and spacecraft to Apophis we need to calculate the total ΔV from LEO (300km). Using the **patched conical approximation** method (Sellers 2005), breaking the trajectory into 3 sections: **Problem 1** moving the spacecraft from LEO to R_{\oplus} just outside Earth's SOI (Sphere of Influence). **Problem 2**- Hoffman transfer from a heliocentric orbit just outside Earth's SOI via an elliptical Hoffman transfer orbit to Apophis at perihelion. **Problem 3**, normally a terminal maneuver to brake the spacecraft from transfer orbit speed at target to an orbit around the target planet. In this case we will not go into orbit around Apophis but just retrofire to reduce speed to match Apophis for a station keeping position. Using the Hoffman transfer orbit is the most economical way

to accomplish this but not the fastest. For these calculations we use two equations based on the conservation of mechanical energy:

$$e = V^2/2 - \mu/R \quad \text{constant for orbit}$$

where e - is *specific mechanical energy* of spacecraft
 V is *spacecraft velocity* km/s
 R is magnitude of *spacecraft/planet radius vector*
 Transposing -- $V = \sqrt{2(\mu/R + e)}$

Also, $e = -\mu/2a$ where a is *semimajor axis* of orbit.

Problem 2 first:

$$\mu_{\text{sun}} \cdot \text{Suns gravitational parameter} = 1.327 \times 10^{11} \text{ km}^3\text{s}^{-2}$$

$$\mu_{\text{Earth}} \cdot \text{Earth's gravitational parameter} = 3.986 \times 10^5 \text{ km}^3\text{s}^{-2}$$

$$a_{\text{Earth}} = 1.496 \times 10^8 \text{ km}$$

$$a_{\text{Apophis}} = 1.3797 \times 10^8 \text{ km}$$

$$a_{\text{transfer}} = (R_{\text{to Earth}} + R_{\text{to Apophis}})/2 = 1.4378 \times 10^8 \text{ km}$$

$$R_{\text{park}} = 6.678 \times 10^3 \text{ km} \quad \text{LEO orbit distance from Earth's center}$$

$$R_{\text{to Earth}} = a_{\text{Earth}} = 1.496 \times 10^8 \text{ km} \quad (\text{assuming circular heliocentric orbit})$$

$$R_{\text{to Apophis}} = 1.3797 \times 10^8 \text{ km}$$

$$e_{\text{Earth}} = -\mu_{\text{sun}}/2a_{\text{Earth}} = -443.5 \quad \text{specific mechanical energy of Earth orbit}$$

$$e_{\text{transfer}} = -\mu_{\text{sun}}/2a_{\text{transfer}} = -461.45 \quad \text{specific mechanical energy of transfer orbit}$$

$$e_{\text{Apophis}} = -\mu_{\text{sun}}/2a_{\text{Apophis}} = -480.90 \quad \text{specific mechanical energy of Apophis orbit}$$

$$V_{\text{Earth}} = \sqrt{2(\mu_{\text{sun}}/R_{\text{to Earth}} + e_{\text{Earth}})} = \mathbf{29.78 \text{ km/s}} \quad (\text{Earth orbital speed})$$

$$V_{\text{transfer@Earth}} = \sqrt{2(\mu_{\text{sun}}/R_{\text{to Earth}} + e_{\text{transfer}})} = \mathbf{29.175 \text{ km/s}}, \quad (\text{initial velocity on transfer orbit})$$

$$V_{\infty\text{Earth}} = |V_{\text{transfer @ Earth}} - V_{\text{Earth}}| = \mathbf{0.6053 \text{ km/s}}. \quad \text{This is } V_1 \text{ required to put the spacecraft onto the transfer orbit from just outside Earth's SOI moving with 0 velocity in the same heliocentric orbit as Earth.}$$

$$V_{\text{transfer@Apophis}} = \sqrt{2(\mu_{\text{sun}}/R_{\text{to Apophis}} + e_{\text{transfer}})} = \mathbf{31.6339 \text{ km/s}} \quad (\text{orbital velocity on arrival at Apophis in transfer orbit.})$$

$$V_{\text{Apophis}} = \sqrt{2(\mu_{\text{sun}}/R_{\text{to Apophis}} + e_{\text{Apophis}})} = \mathbf{31.013 \text{ km/s}}$$

$$V_{\infty\text{Apophis}} = |V_{\text{transfer @ Apophis}} - V_{\text{Apophis}}| = \mathbf{0.6209 \text{ km/s}}. \quad \text{This is retro } V_2 \text{ required to slow the spacecraft to the same speed as Apophis for station keeping.}$$

Problem 1 of the patched conical trajectory would normally take the spacecraft from low Earth orbit (LEO) to R_{-} escaping Earth's SOI on a hyperbolic trajectory moving with 0 velocity at R_{-} . We can combine this burn with V_1 thereby putting the spacecraft on the transfer orbit in one go. Calculating V required to go to R_{-} from a 300km LEO:

$$e_{-}\text{Earth} = (V_{-}\text{Earth}^2)/2 = 0.1832$$

$$V_{\text{hyper @ Earth}} = \sqrt{2(\mu_{\text{Earth}}/R_{\text{park}} + e_{-}\text{Earth})} = \mathbf{10.94 \text{ km/s}}$$

$$V_{\text{park@Earth}} = \sqrt{\mu_{\text{Earth}}/R_{\text{park}}} = 7.725 \text{ km/s}$$

$$\Delta V_{\text{boost}} = |V_{\text{hyper @ Earth}} - V_{\text{park@Earth}}| = 3.215 \text{ km/s}$$

Therefore total V_{boost} takes the interplanetary payload from 300 km LEO to Apophis is: V_2 at Apophis is supplied by the spacecraft propulsion system which is also used for midcourse corrections

Plane Change

The initial parking orbit is normally inclined to the Earth equatorial plane related to the latitude of the launch site, 28.5 for Cape Canaveral and 5.3 for Guiana Space Centre. A plane change would be required for the interplanetary trajectory to align the transfer orbit plane with the orbital plane of the target, in this case Apophis. The angle of the plane change must correct the initial launch angle for the launch site plus the angle of the Earth axis to the ecliptic plus the orbital inclination of the target. So the V_{boost} burn must be corrected to include the total plane change angle. The combined burn from LEO using the cos rule is given by:

$$\Delta V_{\text{combined}} = \sqrt{V_{\text{park@Earth}}^2 + V_{\text{boost}}^2 - 2(|V_{\text{boost}}V_{\text{park@Earth}}|\cos \theta)}$$

The angle θ is the total angle of plane change and defines the direction of the trans-Apophis burn.

For the sake of simplicity assume that the plane change has been completed as part of the orbital insertion by the booster stages and we only require V_{boost} to boost the interplanetary payload into the trajectory that will take it to Apophis from LEO. This is done by the final stage rocket to which the spacecraft payload is attached. To calculate the final stage mass including propellants and payload attached, use rocket equation:

$$\Delta V_{\text{boost}} = I_{\text{sp}} g_0 \ln(m_{\text{initial}}/m_{\text{final}})$$

Assume the final stage propellant is liquid oxygen and liquid hydrogen the Specific impulse is 400 sec. Final mass, m_{final} , is that payload plus empty 3rd stage (tanks and motor say 200kg) is 3853kg. g_0 is 9.8ms^{-2}

Rearranging the rocket equation, the initial mass of the final stage is:

$$\ln(m_{\text{initial}}/m_{\text{final}}) = V_{\text{boost}}/I_{\text{sp}} g_0 = 3215/(400 \times 9.8) = 0.8201$$

$$m_{\text{initial}} = m_{\text{final}} e^{0.8201} = 3853 \times 2.27 = 8746 \text{ kg}$$

Booster Selection

We need to select a booster rocket to put the final stage of total mass of 8991kg into 300 km LEO orbit with the interplanetary payload attached and in the correct orbital plane.

Table 5. Available rocket boosters to LEO for > 10.000 kg

Name	Country	Manufacturer	Payload to LEO
Ariane 5	Europe	Airbus	21,000
Atlas IIIB	USA	Lockheed	10,759
Atlas V	USA	Lockheed	18,850
Delta IV	USA	ULA	23,040
Falcon 9	USA	SpaceX	18.785
H-IIB	Japan	Mitsubishi	16,500
GSLV Mk3	India	ISRO	10,000
Proton	Russia	Khrunichev	23,000
Long March2-3-4	China	CALT	12,000

(Credit WikiBoosters)

There are a lot to choose from. The criteria after performance if reliability and cost. On the basis of reliability and performance fit we chose Atlas IIIB which can lift 10,759kg to LEO (SpaceLaunchReport)

Launch Timing

Time of flight (TOF) is half the period of the transfer orbit:

$$TOF = \frac{1}{2} a^3_{transfer} / \mu_{sun} = 1.4868 \times 10^6 \text{ seconds} = 172.0833 \text{ days}$$

In order to rendezvous with Apophis at the end of the transfer orbit the launch must occur when the Earth is ahead of Apophis in its orbit by a lead angle given by $lead = \omega_{Apophis} \cdot TOF$ in radians. The angular velocity of Apophis in radians/sec

$$\omega_{Apophis} = \frac{1}{2} \mu_{sun} / R^3_{to Apophis} = 2.2478 \times 10^{-7} \text{ radians/s.}$$

$$lead = 1.4868 \times 10^6 \times 2.2478 \times 10^{-7} = 0.3342 \text{ radians} = 19.15 \text{ degrees}$$

The final phase angle $final$ is $180 + lead = 199.15$ degrees. So at the time of launch the Earth should be 199.15 degrees ahead of Apophis. (because Apophis is moving faster on the inner track).

Calculating the initial phase angle:

According to Apophis orbital elements, at epoch 2008 Apr 14.82 Apophis's Mean anomaly (M) was 215.53. Assuming circular orbit and 0 inclination, heliocentric longitude on that day was $M + \lambda + \varpi = 215.53 + 204.45 + 126.54 = 546.52$ or 186.52 degrees. (λ is ascending node, ϖ is argument of perihelion)

The Earth on that day, (2008 Apr 14.82), was 24.82 days past vernal equinox was at $24.82 \times 0.9856 = 24.46$ degrees of heliocentric longitude. Therefore the angular separation between Earth and Apophis, the initial phase angle, $\theta_{initial}$, on 2008 apr 14.82 was $186.52 - 24.46 = 162.06$ degrees.

To calculate the launch date to rendezvous with Apophis we need to calculate time to launch after 2008 April 14.82. This time is given by:

Waiting time = $(\theta_{final} - \theta_{initial}) / (\omega_{Apophis} - \omega_{Earth})$ (Sellers 2005 p249)

where $\omega_{Apophis}$ and ω_{Earth} are angular velocities of the respective objects. $(\omega_{Apophis} - \omega_{Earth})$ is the synodic rate = $1.1127 - 0.9856 = 0.1272$ degrees/day and synodic period is $360/0.1272 = 2830$ days ~ 7.7 years

So waiting time = $(199.5 - 162.06) / 0.1272 = 294.33$ days after 2008 Apr 14.82. Seeing that it is already 2014 Oct we clearly missed that window. The next window is a full synodic period away $293.33 + 2830 = 3123.33$ days after 2008 apr 14.82. Using the Julian date version of the epoch in Table 1: 2454571.31 we add 3123.33 days to get 2457694.64 JD. Using JD to calendar date converter (OnlineConversion) we get **2016 Nov 2** for the launch date. Adding the Time of Flight, 172.0833 days to the launch date we arrive at Apophis on 2457866.72 JD or **2017 April 23**

Mission Timeline

1 Nov 2016	Launch from cape Canaveral to 300km LEO Spend a day adjusting orbit to be coplanar to Apophis
2 Nov 2016	Launch from LEO to trans-Apophis trajectory Some midcourse corrections may be necessary.
23 Apr 2017	Arrive at Apophis . Execute retro burn for V_2 Robotic arms unpack, unfurl and attach solar sail to asteroid. Continue station keeping, monitoring and occasionally adjusting sail attitude to face sun
13 Apr 2029	Apophis flies by Earth at a safe distance.

References:

Ahrens, T. & Harris, A. 1992, Nature 360, 429.
Asher, D. et al. 2005, The Observatory, 125, pp319-322
AEMPweb: <http://aeweb.tamu.edu/aemp/> (retrieved 20/10/14)
Block, J. et al. 2010, Acta Astronautica 68, 984-992
Brown, P. et al. 2013, Nature 503, 238
CSSweb: http://www.lpl.arizona.edu/css/css_mission.html , (retrieved 23 Apr 2014)
Delbo, M. et al. 2007, Icarus 188, p266
Hyland, D. et al. 2009, Texas A & M University
ImpactEffectsProgram: <http://impact.ese.ic.ac.uk/ImpactEffects/> (retrieved 27/10/14)
Ledkov, A. et al. 2014 40th COSPAR Scientific Assembly, Abstract B0.4-93-14
Lobanovsky, Y. 2014 priprint [arXiv:1403.7282](https://arxiv.org/abs/1403.7282)
Margulieux, R. 2011, Thesis, Texas A & M University
McInnes, C. 2004, Planetary and Space Science, 52, pp587-590
McInnes, C. 2001 ESA Bulletin 108
Melosh, H. & Nemchinov, I. 1993, NATURE, 366 p21
NASAweb: <http://neo.jpl.nasa.gov/neo/report.html> (retrieved 23 Apr 2014)
NEOsearch: <http://neo.jpl.nasa.gov/neo/report.html> (retrieved 20/09/14)
NEOdys: <http://newton.dm.unipi.it/neodys/>
NSSweb: <http://www.nss.org/resources/library/planetarydefense/planetarydefense.html>
OnlineConverter: http://www.onlineconversion.com/julian_date.htm
PAN-STARRSweb: <http://pan-starrs.ifa.hawaii.edu/public/home.html> (retrieved 23 Apr 2014)
Sellers, J. 2005, Understanding Space, An introduction to Astronautics, 3rd ed.
SentryWeb1: <http://neo.jpl.nasa.gov/risk/index.html> (retrieved 18/09/14)
SentryWeb2: <http://neo.jpl.nasa.gov/risk/doc/sentry.html> (retrieved 28/10/14)
SpaceGuardWeb: <http://spaceguard.iasf-roma.inaf.it/SSystem/SSystem.html>
SpaceLaunchReport: <http://www.spacelaunchreport.com/atlas3.html>
(retrieved 28/10/14)
WikiBoosters: http://en.wikipedia.org/wiki/Comparison_of_orbital_launchers_families
(retrieved 28/10/14)